Fall 2013 **Math 601** Name: ____________________

Quiz 3

Question 1. (10 pts)
Determine if the given subset is a subspace of the corresponding vector space. (**Show work!**).

(a) (5 pts) The subset of $\mathbb{R}^3$:

$$W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z \geq 0\}$$

Solution: Consider $u = (1, 1, 1)$. By assumption, u is a vector in W. Now take $k = -1 \in \mathbb{R}$. We have

$$(-1)u = (-1, -1, -1),$$

but $(-1) + (-1) + (-1) = -3 < 0$. So $(-1)u$ is not in W.
We conclude that W is not a subspace.

(b) (5 pts) Let $\mathcal{M}_{n\times n}$ be the vector space of all real $n \times n$ matrices.

$$W = \{A \in \mathcal{M}_{n\times n} \mid \mathrm{tr}(A) = 0\}$$

Solution: Recall that

$$\mathrm{tr}(0_{n\times n}) = 0,$$
$$\mathrm{tr}(A + B) = \mathrm{tr}(A) + \mathrm{tr}(B),$$
$$\mathrm{tr}(kA) = k\mathrm{tr}(A).$$

Now use these to verify the conditions for W to be subspace.
We conclude that W is a subspace.

Question 2. (10 pts)

Given $u_1 = (1,0,1)$, $u_2 = (0,1,2)$ and $u_3 = (2,2,6)$ in $\mathbb{R}^3$.

(a) (5 pts) Determine whether u_1, u_2 and u_3 are linearly independent.

> **Solution:** Form the matrix A whose rows are u_1, u_2, u_3:
>
> $$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$
>
> Use Gaussian elimination and get an echelon form of A
>
> $$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$
>
> which implies u_1, u_2 and u_3 are *not* linearly independent.

(b) (5 pts) Determine whether $v = (0,0,1)$ is in the span of $\{u_1, u_2, u_3\}$.

> **Solution:** The idea is similar to Part (a). But this time, form the matrix B whose columns are u_1, u_2, u_3:
>
> $$B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 6 \end{bmatrix}$$
>
> We consider the augmented matrix
>
> $$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 2 & 6 & 1 \end{array}\right]$$
>
> Use Gaussian elimination again and we have
>
> $$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$
>
> which implies the system has no solution. So v is not in the span of $\{u_1, u_2, u_3\}$.